

# INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M-STAT. I YEAR: 2015-16.

## MID SEMESTRAL EXAMINATION

### SAMPLE SURVEYS

Date: 2 February 2016

Max. Score: 40

Duration: 10:20 to 11:45

- Define 'Sampling Design' and 'Sampling Scheme'.
- Show that  $\sum_{j=1}^N \sum_{i=1}^N \pi_{ij} = 2(2-1) + V(2(n))$ , where  $2(n)$  stands for the distinct units in the sample  $s$  and  $2 = E(2(n))$ . Further, show that a lower bound for  $V(2(n)) = f(1-f)$ , where  $2 = [2] + f$ ,  $f$  being the fractional part.
- When does the equality occur in the relation  $\pi_i + \pi_j - \pi_{ij} \leq 1$ .
- Show that  $\sum_{i=1}^N \pi_i = 2$  and explain its practical significance.

$$[2 + (4+2) + 1+2] = [11]$$

- Consider the following population of 4 Zones in Chennai city with the following data on number of degree colleges ( $x$ ) and the amount spent on computer education  $y$  (in hundreds ₹)

Zones	$U_1$	$U_2$	$U_3$	$U_4$
$x$	10	9	12	20
$y$	52	46	61	99

For a pilot survey, it is desired to draw a sample of 2 units (Zones) by probability proportional to  $x$  data and without replacement.

[ p.t.o.

- a) Explain your selection clearly giving reference to random numbers chosen.
- b) Write down Des Raj's ordered estimators (both orders) for  $Y = \sum_{i=1}^4 Y_i$  and the corresponding pooled estimators.
- c) From (b) calculate Lahiri-Murthy's (Symmetrized Des Raj) Estimator.
- d) Using (b) obtain an unbiased estimate of  $V(\hat{Y})$ .  
 $[2 + ((3+1) + (3+1)) + 5 + 2] = [17]$
- 3 a) Write down the Horvitz-Thompson (HT) estimator for the population total  $Y = \sum_{i=1}^N Y_i$ , explaining the symbols used.
- b) Obtain an expression for  $\hat{V}(\hat{Y}_{HT})$  due to Sen-Yates-Grundy.  
 $[2 + 10] = [12]$

Sum	$U_1$	$U_2$	$U_3$	$U_4$
2	10	8	12	08
8	22	46	61	89